

# MODELLING THE ENERGY BUDGET IN THE CITY OF SÃO PAULO: FLAT TERRAIN AND HOMOGENEOUS LAND USE

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## ABSTRACT

An infinity array of street canyons is used to model the diurnal evolution of the energy budget components over an urban canopy in the City of São Paulo. The radiation fluxes on surfaces are modeled according to canyon geometry by considering each canyon infinitely long and oriented parallel to the North-South direction. Street, wall and roof are described using a three-layer model with liquid water uniformly distributed over streets and roofs surfaces. The vertical turbulent sensible and latent heat fluxes are estimated using bulk aerodynamic method. The results indicate that the new model describes appropriately the time evolution of net radiation, albedo, heat fluxes on street, wall and roof, sensible and latent turbulent heat fluxes.

## RESUMO

Um conjunto infinito de cânions é utilizado para modelar a evolução temporal do balanço de energia sobre um dossel urbano da Cidade de São Paulo. Neste modelo os fluxos de radiação são calculados de acordo com a geometria do conjunto de cânions onde cada cânion é infinitamente longo e está orientado paralelo à direção norte-sul. As ruas, paredes e telhados são descritos através de um esquema de três camadas onde a água líquida superficial está uniformemente distribuída somente sobre as ruas e os telhados. Os fluxos verticais turbulentos de calor sensível e latente são descritos através do método aerodinâmico. Os resultados indicam que o novo modelo descreve de forma apropriada à evolução temporal da radiação líquida, albedo, fluxos de calor nas ruas, paredes e telhados e os fluxos turbulentos de calor sensível e latente.

**Keywords:** modelling, energy budget, street canyon, urban canopy.

## INTRODUCTION

Urban boundary layers (UBL) are heterogeneous in the horizontal direction as result of the highly variable land use. In general, most of the urban areas in Brazil are occupied by a set of buildings with a range of size (heights and horizontal dimension) and within-buildings spaces varying enormously. The nature of urban heterogeneity is not restricted to aerodynamic properties associated to the spatial features of the land use but it is strongly dependent on the thermal and

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radiative properties of the elements that form the urban surface. It is very difficult to describe all relevant characteristics of the UBL in numerical and physical models, so that simplifications are always carried out, especially those concerning the geometry of the surface elements (Plate, 1999). Thus, models of urban land use based on street canyons geometry have been built by several researches to overcome those difficulties in numerical and physical models (Masson, 2000; Kastner-Klein *et al.*, 2001; Rotach *et al.*, 2005).

This work describes a numerical model based on an infinity array of street canyons. This model is used to simulate the diurnal evolution of the heat budget components at the surface of an urban canopy located over a flat area in the City of São Paulo.

## STREET CANYON MODEL

The horizontal homogeneity of the urban canopy is imposed by considering the urban surface composed of an infinity array of street canyons, equally spaced and with infinite length. All streets have the same width ( $w$ ) and all buildings the same height ( $h$ ). The street canyon geometry is characterized by the aspect ratio  $h/w$ . In all energy budget simulations described here  $h/w$  is set equal to unity and canyon orientation is parallel to N–S direction.

### *Direct component of solar radiation*

From a geometric point of view, the direct component of the solar radiation at the street,  $S_s$ , building wall,  $S_w$ , and building roof,  $S_r$ , are characterized by the sun zenith angle,  $\lambda$ , and by the aspect ratio of the street canyon as:

$$S_s = \begin{cases} S(1 - \tan \lambda / \tan \lambda_0) & \text{for } |\lambda| < \lambda_0 \\ 0 & \text{for } |\lambda| > \lambda_0 \end{cases} \quad 1$$

$$S_w = \begin{cases} S \sin \lambda & \text{for } |\lambda| < \lambda_0 \\ S \tan \lambda_0 \cos \lambda & \text{for } |\lambda| > \lambda_0 \end{cases} \quad 2$$

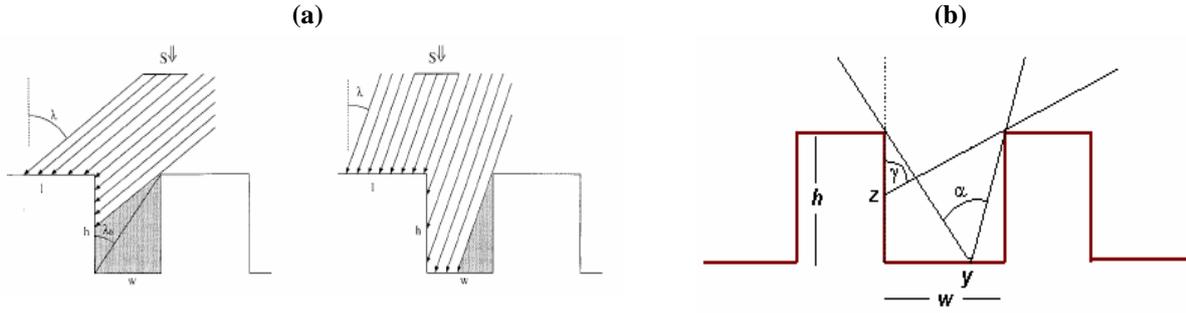
$$S_r = S \cos \lambda \quad 3$$

where  $S$  is the direct component of solar radiation at the surface and  $\lambda_0 = \arctan(w/h)$  is an angle that determines the sun zenith angle for which street is completely shadowed (Fig. 1a). Diurnal evolution of the direct component of solar radiation at the surface,  $S$ , is estimated considering the atmospheric transmissivity,  $\tau$ , constant and equal to 0.73 and the solar radiation at the top of the atmosphere,  $S_0$ , equal to  $1366 \text{ W m}^{-2}$ . The simulations performed here correspond to the summer solstice.

### *Sky view factors*

The diffuse component of solar and downward atmospheric long wave radiation that

effectively reach the shadowed surfaces of the streets and building walls are estimated using the sky view factor (SVF) for a given position at the street,  $\psi_s$ , and at the building wall,  $\psi_w$ .



**Figure 1.** Schematic representation of street canyon geometric parameters for (a) direct solar radiation and (b) sky view factor.

Here, SVF is defined as the ratio of the solid angle for sight field of sky to the solid angle of the entire open field. Due to the fact that the canyon is infinitely long SVF can be estimated in terms two plane angles instead of two solid angles.

The plane angles at points  $y$  and  $z$  on the street and wall, respectively, are indicated in the Fig. 1b. The ratio  $\alpha/\pi$  and  $\gamma/\pi$ , at  $y = w/2$  and  $z = 0$  yields the following SFV,

$$\psi_s = \frac{1}{\pi} \arctan\left(\frac{hw}{h^2 - w^2/4}\right) \quad 4$$

$$\psi_w = \frac{1}{\pi} \arctan\left(\frac{w}{h}\right) \quad 5$$

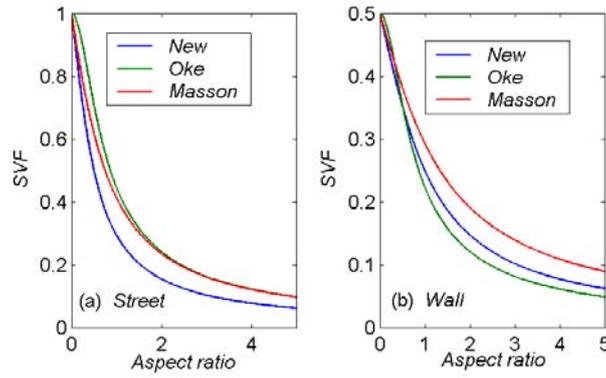
The SVF for street and building wall as a function of  $h/w$  from (4) and (5) are indicated in Fig. 2. For comparison it is also indicated SVF proposed by Oke (1987) and Masson (2000). For  $0 < h/w < 1$  the new  $\psi_s$  and  $\psi_w$  are systematically smaller than Oke and Masson SVF. These differences are less noticeable for  $h/w \gg 1$ , when new  $\psi_s$  and  $\psi_w$  get closer to the ones proposed by Masson 2000.

### ***Diffuse component of solar radiation***

The diffuse solar radiation is obtained by solving the geometric problem of a ray suffering an infinite number of reflections following Masson (2000). The reflections are assumed isotropic and there is not specular reflection in the model.

### ***Downward long wave atmospheric radiation***

The downward long wave radiation emitted by atmosphere is modeled by using the Brunt formula (Bárbaro *et al.*, 2006). It is supposed a constant pressure vapor of 17 mb. Long wave radiation within the canyon is modeled by taking into account only *one* re-emission. The emissivity utilized for street, wall and roof are indicated in table 1.



**Figure 2.** Sky view factor for (a) street and (b) building wall, based on the new formulation, Oke (1987) and Masson (2000).

### ***Heat flux in the solid elements of the surface***

The heat exchange in the canyon rigid frontiers satisfies the energy conservation principle at surface. The input is net radiation so that  $G = H + LE - R_n$ ,  $G$  is heat flux in the solid elements of the surface,  $H$  and  $LE$  are the turbulent sensible and latent heat fluxes and  $R_n$  is the net radiation flux at the surface. Here, upward non-radiative fluxes and downward radiative fluxes are positive.

<b>Table 1. Radiative properties of the street, building walls and roof.</b>			
<b>Properties</b>	<b>STREET</b>	<b>BUILDING WALL</b>	<b>BUILDING ROOF</b>
Emissivity	0.94	0.85	0.90
Albedo	0.08	0.15	0.25

Following Mason (2000), the solid surface elements are divided in three layers (Fig. 3). Energy equation is solved using the Euler finite difference scheme described in (5) and (6). Heat flux at the lowest layer ( $G_4$ ) is set equal to zero for the streets. Temperature at the street lowest layer and at the inner wall and roof building layers are set constant,  $T_{in}$ , and equal to 20 °C. All layers properties (albedo, emissivity, heat conductivity,  $\kappa_T$ , heat capacity,  $C$ , and layer thickness,  $\delta$ ) are indicated in Tables 1 and 2. All inner layers are considered dry, as well as the first wall layer. The equation of heat flux within the rigid frontiers is then written as:

$$C_k \frac{\partial T_k}{\partial t} = \frac{1}{\delta_k} (G_k - G_{k+1}), \quad 5$$

$$G_{k+1} = -2 \kappa_{Tk} \left( \frac{T_k - T_{k+1}}{\delta_k + \delta_{k+1}} \right), \quad 6$$

where  $\delta_k$  is the thickness of the  $k$ th-layer and  $G_1 (= H + LE - R_n)$  is the heat flux into the outer layer. It is used *zero-flux boundary condition* for the street bottom layers. Boundary condition for inner wall and roof layers are given by  $T_4 = T_{in}$ . Initial layer temperatures are indicated in Table 2.

### ***Turbulent fluxes***

Sensible and latent turbulent heat fluxes are estimated using bulk formulas and the temperature difference between each rigid surfaces and the adjacent air. It is assumed the same value of Stanton number,  $C_H$ , for both sensible and latent turbulent fluxes. The bulk turbulent fluxes

are estimated adding up the contribution of individual fluxes weighted by surface area occupied by street, roof and walls.

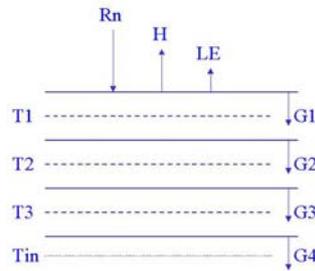


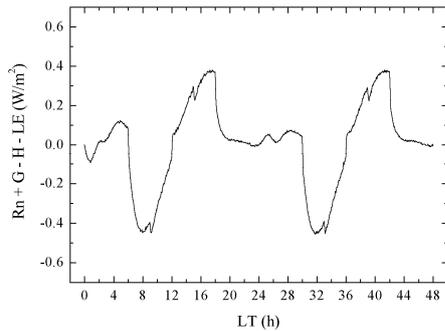
Figure 3. Three-layer scheme for heat flux into the rigid surfaces.

Table 2. Thermal properties of the street, building walls and roof.					
STREET					
Layer k	Thermal conductivity ( $\text{m}^2 \text{s}^{-1}$ )	Heat capacity ( $\text{J K}^{-1} \text{m}^{-3}$ )	Layer thickness (m)	Initial Condition	
				Temperature ( $^{\circ}\text{C}$ )	Moisture content
1	0.50	$1.42 \times 10^6$	0.20	25	0.025
2	0.50	$1.42 \times 10^6$	0.40	22	0
3	0.50	$1.42 \times 10^6$	1.00	20	0
BUILDING WALL					
Layer k	Thermal conductivity ( $\text{m}^2 \text{s}^{-1}$ )	Heat capacity ( $\text{J K}^{-1} \text{m}^{-3}$ )	Layer thickness (m)	Initial Condition	
				Temperature ( $^{\circ}\text{C}$ )	Moisture content
1	0.50	$1.42 \times 10^6$	0.05	25	0
2	0.50	$1.42 \times 10^6$	0.10	22	0
3	0.50	$1.42 \times 10^6$	0.05	20	0
BUILDING ROOF					
Layer k	Thermal conductivity ( $\text{m}^2 \text{s}^{-1}$ )	Heat capacity ( $\text{J K}^{-1} \text{m}^{-3}$ )	Layer thickness (m)	Initial Condition	
				Temperature ( $^{\circ}\text{C}$ )	Moisture content
1	0.50	$1.42 \times 10^6$	0.05	25	0.025
2	0.50	$1.42 \times 10^6$	0.20	22	0
3	0.50	$1.42 \times 10^6$	0.05	20	0

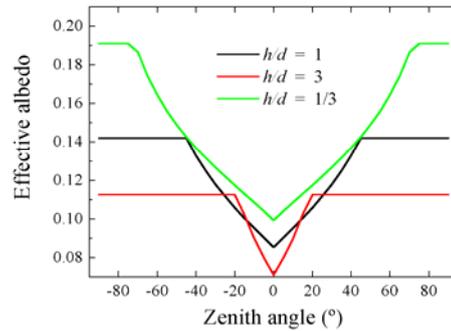
## RESULTS AND CONCLUSION

The model inputs are: (i) global solar radiation; (ii) air temperature; (iii) air specific humidity; and (iv) wind velocity at the reference level fixed at  $2h$ . It was assumed that 15% of global solar radiation is diffuse. Such assumption is based on observational data for clear sky conditions in São Paulo city (Oliveira *et al.*, 2002). The air properties are described as a function of time according observations (Oliveira and Soares, 2004). The parameters used to initialize the simulations are described in Table 2.

In Fig. 4 is shown the residue of the energy budget at the surface, simulated for two days. The incoming solar radiation is set at  $1000 \text{ W/m}^2$ . The greatest difference occurs about midday and is not greater than  $0.5 \text{ W/m}^2$ , which corresponds to a negligible amount of the total input energy over the canyon.



**Figure 4.** The total residual budget.



**Figure 5.** Canyon effective albedo for some aspect ratios.

Fig. 5 indicates the effective albedo for three aspect ratio: 1/3, 1 and 3. It is assumed that the reflections on an individual surface are isotropic, so that their albedoes are independent of zenith angle. Although individual surfaces have albedos independent of zenith angle, the effective albedo varies with sun position. The effective albedo becomes constant when  $|\lambda| > \lambda_0$ , where  $\lambda$  is the zenith angle and  $\lambda_0$  is given by  $\arctan(w/h)$ .

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