IL NUOVO CIMENTO VOL. 15 C, N. 4 Luglio-Agosto 1992

Multiple Master Length Scales for Stable Atmospheric Boundary Layer.

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(ricevuto il 20 Agosto 1991; approvato l'11 Febbraio 1992)

Summary. – A new method of derivation of characteristic turbulence length scales is analysed in a procedure that seems to be a more adequate way of determining stable boundary layer master length scales. The novel feature of this derivation is a provision for *multiple* length scales, one for each different spacial direction. In its general formulation, these multiple master length scales show a form similar to earlier proposals by Blackadar and his followers.

PACS 92.60.Fm – Boundary layer structure and processes.

1. – Introduction.

Under clear skies and over land, the nocturnal Stable planetary Boundary Layer (SBL) is characterized by negative values of turbulent vertical fluxes of sensible heat, resulting from the wind-shear–induced turbulence and the stable thermal stratification produced by the radiational cooling of the surface.

Differently from the diurnal Convective Boundary Layer (CBL) with typical height of two kilometers, the SBL vertical extension is of only a few hundred meters. The source of the turbulent kinetic energy in the latter case (mechanical only) is much less intense than in the former case (thermal and mechanical) and it is strongly depleted by the work done by the turbulence against the stable stratification. At middle latitudes, the SBL wind varies with height, both in direction and magnitude. These variations depend on the intensity of the forcing (horizontal pressure gradients), the actual thermal stability and the intensity of the previous CBL. The presence of low-level jets at the top of the SBL is very common and intrinsically related to their dynamics [1, 2].

Very often, the thermal stability is so strong that the turbulent motions are inhibited, causing the turbulence to partly subside and become intermittent. Due to the unsurmountable difficulties inherent to this state of the turbulence, we restrict our analysis and its applicability to a fully turbulent SBL. Even a fully turbulent SBL responds to changes in the surface forcing slowly, so that it is not in equilibrium with the surface. Therefore its statistical structure is better described by a local similarity [2]. For instance, the turbulent transport of passive contaminant throughout the SBL is described by a local Obukhov's length scale [3].

The importance of turbulent transport of momentum, heat, water vapour and contaminants was identified a long time ago, as a major goal in the study of the SBL. A simplified and physically reasonable way to describe this transport has had a recognized impact in the prediction of low-level jets, fog and frost formation and air quality. The basic equations for describing these processes are well known but their full utilization is not operational both from analytical and numerical viewpoints. In practice one has to resort to some kind of shortcuts represented by closure models and associated parameterization of higher-order moments. Moreover, even after the most simplifying assumptions, the resulting equations are not suited for purely analytical treatment. However these simplified models are easily amenable to numerical analysis, mainly in recent times when computer speed and memory size become significantly larger.

The second-order closure models have been claimed to be one of the most appropriated tools to simulate numerically the planetary boundary layer. They reconcile the numerical amenability with a more basic physical description of the turbulent process. Several second-order closure models simulate appropriately the CBL. Simulating the SBL on the basis of a second-order closure model is a more difficult task. This certainly explains why SBL has been numerically simulated less frequently than the corresponding CBL [4]. The worst difficulty has to do with the proper choice of a *turbulent master length scale* [5] that leads to an adequate parameterization of the undetermined terms in the equations for the evolution of the second-order moments. It is also emphasized in the literature [4] that a weakness common to all such closure models is rooted in the correct choice of characteristic lengths and on the fact that it is admitted that only one length scale would be generically sufficient for an adequate description of turbulent transport phenomena. Some authors [6] even doubt about the possibility of determination of characteristic length scales suited for any general purpose. However, some length scales such as Blackadar's length scale [7] have acquired an almost universal acceptance. These seemingly contrasting facts indicate, in our believe, the necessity of more general criteria for supporting the proper selection of the referred length scales.

In this paper we bring not only a more natural derivation of length scales adequate to geophysical boundary layers but, most importantly, we introduce the concept of multiple length scales as opposed to the current usage of a single length scale to describe transport in all three spacial directions. To support the new concept we compare parameters derived from this theory to experimental and/or empirical data found in the literature and come to suprisingly good results.

This paper is organized as follows: in sect. **2** we discuss Blackadar's and Brost-Wyngaard's length scales and derive new length scales from considerations about the turbulent energy spectrum; in sect. **3** we utilize experimental data to compare the new length scales with that proposed by Brost and Wyngaard; finally, in sect. **4** we present additional comments and suggest possible applications for multiple length scales, mainly in connection with the development of a more realistic direct numerical simulation of geophysical boundary layers.

2. – Turbulent length scales.

A widely used length scale is Blackadar's [7] length scale l expressed by

$$\frac{1}{l} = \frac{1}{\kappa z} + \frac{1}{l_0}, \tag{1}$$

where κ is von Karman's constant, z is the height above the ground and l_0 is the scale characteristic of the energy containing eddies. Because Blackadar's scale is employed in the derivation of characteristic length for transport phenomena in all three different spacial directions it is generally referred to as the *master length scale*. As easily seen from [1], the master length scale is proportional to z near the ground and goes as l_0 at the top of the planetary boundary layer when z is large.

One crucial point about the use of the master length scale is the modeling of the size of the energy containg eddies l_0. In this respect several procedures are currently found in the literature [8].

In a well-known case, the Brost-Wyngaard model [9], the characteristic length $l_0 = l_B$ is determined by the balance between inertia forces and buoyant forces by the relation

$$l_B = C\frac{\sigma_w}{N}, \tag{2}$$

where N is the Brunt-Väisälä frequency determined by

$$N = \left(\frac{g}{T_0}\frac{\partial \Theta}{\partial z}\right)^{1/2}; \tag{3}$$

σ_w is a characteristic velocity scale determined by the variance of the vertical turbulent velocity and $C \equiv 1.69$ is also an empirical constant suggested by Brost and Wyngaard [9] and chosen as to yield critical flux and gradient Richardson numbers close to 0.2 and 0.25, respectively. In eq. (3) g is the acceleration of gravity, T_0 is a reference temperature and Θ is the potential temperature.

A possibility for modeling l_0 might be based on the local similarity structure of the SBL and its related energy spectrum, keeping Brost and Wyngaard's interpretation of l_0 as the size of the energy containing eddies. Subsequent theoretical works [2,10,11] on second-order correlation approximation and related closure models, have also exhibited similarity structures for this regime. This possibility has recently been used to model the energy spectrum of the SBL starting from basic physical arguments [12].

Nevertheless this similarity structure, at least for the temperature, only occurs after high-pass filtering of the data time series [3]. The nonfiltered temperature data, collected at Cabauw, exhibited a much different behaviour than filtered data. In order to deal with the local character of the turbulence, it was first assumed that the appropriated flux scales should depend on local values of the Reynolds stresses $\tau(z)$, on the vertical heat flux $\overline{w\theta}(z)$ and on the local Monin-Obukhov length Λ, as defined by

$$\frac{\tau}{\tau_0} = (1 - z/h)^{\alpha_1}, \tag{4}$$

$$\frac{\overline{w\theta}}{\overline{w\theta}_0} = (1 - z/h)^{\alpha_2} \tag{5}$$

and

$$\frac{\Lambda}{L} = (1 - z/h)^{3\alpha_1/2 - \alpha_2}. \tag{6}$$

In (4)-(6), $\tau_0 \equiv \rho u_*^2$ is the surface stress, ρ is the air density, L is the Monin-Obukhov length and α_k are constants to be determined by fitting of the model to experimental data. It is also expected that these hypotheses be applicable in stable regimes over homogeneous terrains where turbulence can be treated as continuous and not dominated by gravity waves.

Recent development on local similarity theory [13] has lead to the derivation [3] of a local energy spectrum

$$\frac{nS_i(n)}{U_*^2} = 1.5\, C_i \frac{(C_\varepsilon)^{2/3} f/q}{\left[f_i^{5/3} + 1.5(f/q)^{5/3}\right]}, \tag{7}$$

where $i \doteq u, v, w$, $U_* \equiv \sqrt{\tau/\rho}$, f_i is the frequency of the spectral peak in the neutral stratification, $C_\varepsilon = 1.25$ is an empirical constant proposed by Sorbjan [13], $f = n\,z/U$ is the reduced frequency derived from the absolute frequency n and the mean wind speed U, f_i is a reference frequency derived in a similar way from n_i , the frequency at the maximum on the spectrum. Also q is a similarity function given by

$$q = 1 + 3.7\frac{z}{\Lambda}, \tag{8}$$

with parameters determined by Sorbjan [13] that describes the maximum energy eddy frequencies along the vertical and C_i is a constant derived from Kolmogorov law for the inertial subrange.

The integral of S_i over all frequency range yields the variance for the velocity components. Therefore (7), after one integration in frequency, leads to

$$\sigma_i^2 = 2.7\, C_i \frac{U_*^2}{f_i^{2/3}}. \tag{9}$$

We now insert σ_i back in eq. (7) and obtain

$$\frac{nS_i(n)}{\sigma_i^2} = 0.64 \frac{f_i^{2/3}}{\left[f_i^{5/3} + 1.5(f/q)^{5/3}\right]} \frac{f}{q}. \tag{10}$$

The value of the spectrum at the origin as given by a statistical diffusion theory [3] defines an eddy diffusivity and therefore $S_i(0)/\sigma_i^2$ is one typical turbulent time scale for transport processes. The limit of S_i for $n \to 0$ yields

$$\frac{S_i(0)}{\sigma_i^2} = \frac{l_i}{U}. \tag{11}$$

This procedure yields a characteristic length for turbulent transport processes that has Blackadar and Delage's [8] form

$$\frac{1}{l_i} = \frac{1}{(0.64/f_i)z} + \frac{1}{(0.17/f_i)\Lambda}. \tag{12}$$

We stress that (12) has been derived from a detailed parameterization of the energy spectrum which involved the characteristic time scale for eddy transport. Most important to notice is that in this case we obtain three length scales instead of the usual single form. Each component of this *multiple master length scale* applies to a different direction. Therefore, the turbulent transport is governed by a specific length scale in each different spatial direction.

For completeness we now specify these scales for all three directions taking the value of the frequency of the maximum energy containing eddy in the neutral case f_i from the experiment as analysed by Sorbjan [13]. In this case study, it was shown that $f_w = 0.33$, $f_u = 0.058$ and $f_v = 0.22$ for series considered in the experiment. Other authors may come to different values but qualitatively they do not really differ [14]. The substitution of these values of f_i into eq. (12) yields

$$\frac{1}{l_u} = \frac{1}{11.03\,z} + \frac{1}{2.93\,\Lambda}, \tag{13}$$

$$\frac{1}{l_v} = \frac{1}{2.91\,z} + \frac{1}{0.77\,\Lambda}, \tag{14}$$

$$\frac{1}{l_w} = \frac{1}{1.94\,z} + \frac{1}{0.52\,\Lambda}. \tag{15}$$

It is interesting to notice that, with respect to the vertical length scale, the theoretical factor 1.94 found in this derivation was previously determined from experimental data [15] as equal to 2. This certainly is a surprisingly good approximation. It is also important to stress at this point that this derivation of the (multiple) master length scale leads simultaneously to the determination of the characteristic size of the energy containing eddies l_{0i} which in the present case has different values for each spacial direction (anisotropic transports) and is determined in terms of the *local* Monin-Obukhov length Λ. Therefore in any second-order closure model study, it is fundamental to take into account the spacial dependence of Λ, a point already stressed in the literature [2].

3. – Comparison of different models.

Two important characteristic lengths applicable to the stable boundary layer are the local Obukhov length Λ and the vertical displacement length σ_w/N which directly connect to other characteristic lengths. As mentioned before, the crucial point about applying the master length scale is the proper choice of size l_0 for the energy containing eddies. For this reason it is now important to compare, for example, the result of Brost and Wyngaard (2) with the term containing Λ in eq. (12). This analysis is basically a generalization of a procedure already developed for the surface layer case [16]. To this end the hypothesis that the form of the similarity functions in the outer layer should be identical to the form of the Monin-Obukhov similarity functions in the surface layer [10] is adopted. The similarity function Φ_H for the heat flux generalized for a SBL is

(16)
$$\Phi_H = \frac{\kappa z}{t_*}\frac{\partial \Theta}{\partial z}, \qquad = 0.74 + 4.7\frac{z}{\Lambda}$$

where $t_* = -\overline{w\theta}/U_*$ is a characteristic temperature scale. The last equality in eq. (16) is an empirical relation derived from the data obtained in the Kansas [17, 18] experiment. Equation (16) can be solved for t_*

(17)
$$t_* = -\frac{\kappa z}{0.74 + 4.7\, z/\Lambda}\frac{\partial \Theta}{\partial z},$$

and the kinetic heat flux can be expressed as

(18)
$$\overline{w\theta} \equiv -t_* U_* = -\frac{\kappa z U_*}{0.74 + 4.7\, z/\Lambda}\frac{\partial \Theta}{\partial z}.$$

The stable stratification inhibits vertical motions and consequently reduces the turbulent length scale. When this scale becomes much smaller than the height above the ground the structure of the turbulence does not respond to the ground conditions and as a consequence the explicit dependence on z is lost. This situation is commonly known as the *z-less* stratification [19]. Therefore, for large z/Λ, the heat flux approaches

(19)
$$\overline{w\theta} = -\frac{\kappa}{4.7}\Lambda U_* \frac{\partial \Theta}{\partial z}.$$

We now insert the definition of local Obukhov length

(20) $$\overline{w\theta} \equiv -\frac{\Theta}{\Lambda}\frac{U_*^3}{\kappa g},$$

into eq. (19) and obtain

(21) $$\Lambda = \frac{2.17}{\kappa}\frac{U_*}{N}.$$

To proceed, we use (21) together with relations (2), (15) and $\sigma_w = BU_*$ in order to obtain

(22) $$\frac{1}{l_\mathrm{B}} = \frac{1}{C(\sigma_w/N)} = \frac{1.92}{\Lambda},$$

or

(23) $$B = \frac{1.13}{\kappa C},$$

for von Karman's constant $\kappa \approx 0.4$.

An experimental average value of B may be estimated from measureaments of σ_w carried on stable boundary layers. Sorbjan [13] estimated $\sigma_w \approx 1.6U_*$ from Minnesota data. Data collected at Cabauw [2] lead to a similar value with $\sigma_w \approx 1.5U_*$. Other more recent measuraments made on BAO - Tower [20] and SESAME experiment [21] yield, respectively, $\sigma_w \approx 2U_*$ and $\sigma_w \approx 1.73U_*$. The substitution of $C = 1.69$ into eq. (23) leads to $B = 1.67$, a value that replicates the experimental results with high accuracy. The values of the Richardson numbers that lead to this value of C were obtained by Nieuwstadt from Cabauw tower experiment under conditions of rather strong wind and continued turbulence.

Another interesting physical relation dependent on the Richardson number is the ratio between the vertical displacement length σ_w/N and the local Obukhov length Λ. By use of (22) we find that the above ratio is $\sigma_w/N\Lambda \approx 0.31$. At this point it is worth mentioning that a recent large eddy simulation [22] has lead to values for this ratio in the range $0.30 \div 0.35$, in good agreement with the present model.

4. – Conclusions.

In this paper we presented a method of derivation of characteristic turbulence length scales. This was achieved by first presenting what is proposed as a more adequate way of determining a stable boundary layer master length scale. The novel feature of this derivation is the provision for *multiple* master length scales, one for each different spacial direction. In its general formulation, the multiple master length scales l_i show a form similar to that proposed earlier by Blackadar [7], Brost and Wyngaard [9] and Venkatram *et al.* [16]. An interesting feature of these multiple characteristic length scales l_i is that they become, like Delage's length scale, proportional to the local Monin-Obukhov length on the upper part of the SBL. The nature of the subject is not suited for a direct check between experiment and model. However a few aspects and consequences of the model could be easily checked against known values of empirical and/or experimentally measured parameters. One such test was the comparison of the resulting energy containing eddy size as proposed by Brost and Wyngaard and the result of the present model. In addition, a comparison of the models ratio of characteristic lengths $\sigma_w/N\Lambda$ with the same parameter as drawn from a large eddies simulation, definitely qualifies this spectral method as a sound candidate for an alternative procedure to improve direct numerical simulation of the nocturnal stable planetary boundary layer. A numerical simulation study of the SBL is presently carried out in which these scales are implemented in a second-order closure model. The preliminary results have indicated that the time and space evolution of the SBL dynamical and thermodynamical structure is somehow sensitive to the use of master length scales proposed herewith. The complete results of these simulations will be published shortly elsewhere.

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This work has been partially supported by the Brazilian agencies Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq), Financiadora de Estudos e Projetos (FINEP), Fundação de Amparo a Pesquisa do Estado de São Paulo (FAPESP) and Fundação de Amparo a Pesquisa do Estado do Rio Grande do Sul (FAPERGS).

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